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* Solve the following recurrence relation using master method .

T(n) = 2T(n/2) + cn

Given recurrence relation: T(n)=2T(n/2) + cn

T(n) = aT(n/b) + f(n) , where a >= 1, b > 1

a = 2 , b= 2 , f(n) = cn

T(n) = 2T(n/2) + cn

Since 𝑓(𝑛) = 𝑐𝑛 is asymptotically equal to *n^*log*b*​*a*=*n*,

we fall into case 2 of the Master Theorem.

*T*(*n*)=Θ(*n^*log*b*​*a*log*n*)

*T*(*n*)=Θ(*n*log*n*)

So, in this case, the solution to the recurrence relation 𝑇(𝑛)=2𝑇(𝑛/2)+𝑐𝑛 using Master Theorem Case 2 is *T*(*n*)=Θ(*n*log*n*). This means that the time complexity of the algorithm described by this recurrence relation is bounded asymptotically by Θ(*n*log*n*), indicating that the algorithm's time complexity grows at the rate of *n*log*n* as *n* increases.

* *Let's verify the solution ( T(n) = Theta(n log n) ) using the substitution method.*

*We assume that ( T(n) = an log n + bn ) and then substitute it back into the original recurrence relation.*

*1. \*\*Assumption:\*\**

*We assume that ( T(n) = an log n + bn ).*

*2. \*\*Substitution:\*\**

*[ T(n) = 2T(n/2) + cn ]*

*[ an log n + bn = 2[a(n/2) log(n/2) + b(n/2)] + cn ]*

*3. \*\*Simplify:\*\**

*[ an log n + bn = 2[a(n/2) log(n/2) + b(n/2)] + cn ]*

*[ an log n + bn = 2[a(n/2) log n - alog 2 + b(n/2)] + cn ]*

*[ an log n + bn = 2[a(n/2) log n - alog 2 + bn/2] + cn ]*

*[ an log n + bn = a(n log n - alog 2 + n/2 log n) + bn - alog 2 + cn ]*

*4. \*\*Match Coefficients:\*\**

*- Coefficients of ( n log n ): ( a = 2a ), so ( a = 2 ).*

*- Coefficients of ( n ): ( b = b/2 - alog 2 + c ), so ( b/2 - alog 2 + c = b), and ( c = b2 - alog 2 ).*

*5. \*\*Verify Conditions:\*\**

*- We need to verify that ( c) is positive to ensure that our assumption of( T(n) ) is correct.*

*c = b/2 – alog 2 = b/2 - 2log 2*

*Since b is a constant and log 2 is a constant, c will be positive if b is sufficiently large.*

*6. \*\*Conclusion:\*\**

*Since we found values for a , b , and c that satisfy the recurrence relation, and c is positive for sufficiently large b , our assumption that ( T(n) = an log n + bn is correct.*

*Therefore, by the substitution method, we have verified that ( T(n) = Theta(n log n) ) is indeed the solution to the recurrence relation ( T(n) = 2T(n/2) + cn ).*